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Effects of a finite electric field on the localization in anisotropic two-dimensional systems

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Abstract. The direct influence of a d.c. electric field on weak localization is investigated on the basis of a quantum diffusion model for an anisotropic two-dimensional electron gas. The anisotropy is treated both in a model of anisotropic effective masses and in a tight-binding model for parallel chains. The crossover of the delocalization edge from a power-law 1D-like dependence to an exponential decrease, characteristic for a 2D system, is investigated. The appearance of a delocalization edge gives rise to a long-time power-law current relaxation according to $j(t) \sim 1/\sqrt{t}$, independently of the anisotropy.

1. Introduction

The nature of the electronic states in a two-dimensional random potential has received a great deal of attention. It turns out that the localization length depends exponentially on the dimensionless disorder parameter $k_F\lambda_0$ (k_F is the Fermi momentum and λ_0 the elastic scattering length). Any measurement of the resistance involves a finite electric field, which may give rise to a further increase of the localization length in the non-linear electric field regime. Therefore, it is interesting to study how the electric field affects the localization in two dimensions and whether there is a delocalization edge or not. Over the years this has been a quite controversial issue. On the one hand it is stated that a d.c. electric field does not break the time-reversal invariance and, therefore, as long as heating effects can be neglected, the electric field has no influence on weak localization [1, 2, 3]. By contrast, Kirkpatrick [4] generalized the self-consistent theory of Anderson localization [5, 6] to include the effect of a uniform electric field and concluded that Anderson localization is not possible in a disordered two-dimensional system ($d = 2$) at finite electric fields. Similarly, Tsuzuki [7] and Kaveh *et al* [8] predicted a direct influence of a d.c. electric field on the quantum interference, which is described by the so-called Cooperon. There are experimental results which seem to support both sides [9, 10].

The situation is quite different for the case of a one-dimensional disordered solid in a finite electric field. It is intuitively clear that, for a given degree of randomness, the spatial range of localized states is much greater in two dimensions than in one dimension. Concerning the electric field dependence Prigodin [11] obtained exact results by extending the Berezinskii diagrammatic technique. According to this approach power-law localization emerges for weak fields, while for higher fields a mobility edge appears, above which the states are extended. The experimental verification, e.g. on quantum wires, is complicated by Joule heating, which will invalidate the assumptions of the model unless the carrier density is very small and the electric field is not too large.

Within a self-consistent approach of quantum diffusion we [12] reproduced the exact result for the $d = 1$ delocalization field strength and predicted a delocalization edge for the two-dimensional case too, which, however, depends exponentially on the disorder parameter $k_F\lambda_0$. The appearance of this mobility edge leads to a long-time tail in the current relaxation, which was treated in [13]. Whereas very low carrier concentrations and extremely low lattice temperatures are necessary for measuring the predicted $d = 2$ effects, we found that the requirements for the one-dimensional case are far less restrictive. With respect to the direct influence of a constant electric field on weak localization, both the experimental and theoretical situation are so different for $d = 1$ and $d = 2$ systems that it should be useful to consider the intermediate case of an anisotropic two-dimensional electron gas. The aim of this paper is to provide a theoretical model for studying the constant-electric-field-mediated crossover behaviour of the delocalization edge and the related implications for the current relaxation.

2. The delocalization edge

According to the theory of Anderson localization proposed by Vollhardt and Wölfle [5, 6] the dynamical diffusion coefficient D is derived from the self-consistent equation

$$\frac{D_0}{D} = 1 + \frac{1}{\hbar\pi N_F} \sum_k C(\boldsymbol{k}) \tag{1}$$

where $D_0 = v_F^2\tau_0/d$ is the bare diffusion coefficient in a d-dimensional lattice (v_F is the Fermi velocity, τ_0 the elastic scattering time and N_F the density of states at the Fermi surface). For an anisotropic two-dimensional electron gas two components D_x and D_y of the diffusion coefficient appear. It has been shown in [14, 15] that, near the weak-coupling fixed point of the scaling theory of localization, the effects of anisotropy can be completely incorporated into only one diffusion coefficient. We adopt this treatment here and introduce the following anisotropy parameter $\alpha^2 = D_y/D_x$. The Cooper propagator $C(\boldsymbol{k})$ is the solution of the Laplace-transformed quantum diffusion equation, the form of which is equivalent to the continuity equation of the electrical current. As in [12, 13] we consider the restricted electric field region where it can be assumed that the Einstein relation between the renormalized mobilities $\mu_{x,y}$ and the diffusion coefficients $D_{x,y}$ is still valid:

$$\mu_{x,y} = e\frac{N_F}{N} D_{x,y}. \tag{2}$$

($N = 2N_F\varepsilon_F/d$ is the total electron concentration.) The Laplace-transformed autocorrelation function of the Cooperon has the following form

$$\sum_k C(\boldsymbol{k}) = \sum_k \frac{1}{s + D_x k_x^2 + D_y k_y^2 + (e^2N_F^2/4N^2)(D_x E_x^2 + D_y E_y^2)} \tag{3}$$

where the electric field is aligned along the x–y plane $\boldsymbol{E} = E(\cos\phi, \sin\phi, 0)$. In the thermal equilibrium the parameter s of the Laplace transformation can be identified by the complex frequency $-i\omega$, which may also be replaced by $1/\tau_\varepsilon$, where τ_ε is the inelastic scattering time. The integral in (3) diverges at the upper boundary and one has to introduce appropriate cut-off wave vectors to get finite results. These boundaries are due to the fact that the diffusion picture is applicable only for large spatial regions. However, the main contribution in the integral (3) comes from small k_x- and k_y-values, so the upper cut-off wave vectors κ_x and κ_y may be determined by physical reasoning. According to our one-parameter approach with respect to the anisotropy we introduce two characteristic cut-off parameters by $\beta^2 = \kappa_y/\kappa_x$ and $\tilde{\kappa}^2 = \kappa_x\kappa_y$. The integrals in (3) are elementary and one can easily express the autocorrelation function in terms of the renormalized diffusion coefficient $D = \sqrt{D_x D_y}$

$$\sum_k C(k) = \pi \ln\left[\frac{2\sqrt{\gamma^2+1+h\gamma}+2+h\gamma}{\gamma(2+h)}\right] \tag{4}$$

where

$$\gamma = \frac{1}{\tilde{\kappa}^2}\left(\frac{s}{D} + \frac{e^2N_F^2}{4N^2}\left(\frac{E_x^2}{\alpha} + \alpha E_y^2\right)\right) \tag{5}$$

and

$$h = \alpha\beta^2 + \frac{1}{\alpha\beta^2}. \tag{6}$$

Because our approach is restricted to low electric fields ($\gamma \ll 1$) we can further simplify (4) and obtain from (1) the following self-consistent equation:

$$D = D_0 - \frac{1}{2\pi^2\hbar N_F}\sinh^{-1}\left(\frac{s}{D\kappa^2} + \varepsilon^2\right)^{-1/2} \tag{7}$$

with the dimensionless parameter of the electric field

$$\varepsilon = \frac{eEN_F}{2\kappa N}\left(\frac{1}{\alpha}\cos^2\phi + \alpha\sin^2\phi\right)^{1/2}. \tag{8}$$

The self-consistent equation (7) for the renormalized dynamical diffusion coefficient is our main result, which depends only on one effective cut-off parameter $\kappa = \tilde{\kappa}/(\sqrt{\alpha}\beta + 1/\sqrt{\alpha}\beta)$. Furthermore, $D_0 = \sqrt{D_{0x}D_{0y}}$ is the bare effective diffusion coefficient of the anisotropic system, which is expressed in terms of an isotropic relaxation time τ_0 and averaged velocities along the x and y directions, respectively

$$D_{0x,y} = \tau_0\langle v_{x,y}^2\rangle \qquad \langle v_{x,y}^2\rangle = \frac{1}{N_F}\int \mathrm{d}^2k\left[\frac{1}{\hbar}\frac{\partial\varepsilon(k)}{\partial k_{x,y}}\right]^2\delta(\varepsilon_F - \varepsilon(k)). \tag{9}$$

$\varepsilon(k)$ is the two-dimensional energy spectrum. It is in line with our one-parameter picture of the anisotropy that there is only one renormalized scattering time τ, defined by $D_{x,y} = \tau\langle v_{x,y}^2\rangle$. The effects of anisotropic relaxation times were investigated in [16].

At the delocalization edge E_0 the dynamical diffusion coefficient vanishes (if $s = 0$) and one derives from (7) the simple expression

$$\varepsilon_0 = \frac{eE_0N_F}{2\kappa N}\sqrt{\frac{1}{\alpha}\cos^2\phi + \alpha\sin^2\phi} = \frac{1}{\sinh\Theta} \tag{10}$$

with

$$\Theta = 2\pi^2\hbar N_F D_0\,. \tag{11}$$

It is obvious that this result for the delocalization edge field strength exhibits a crossover behaviour from a power-law dependence $\varepsilon_0 = 1/\Theta$ at $\Theta \ll 1$, which is characteristic for one-dimensional systems, to a steep exponential decrease $\varepsilon_0 = 2\exp(-\Theta)$ at $\Theta > 1$, expected for two-dimensional electron gases [12]. This crossover gives rise to some peculiarities in the current relaxation, too.

3. Current relaxation

Next we consider the time-dependent current, which results if initially at the time $t = 0$ a constant electric field is switched on. According to the Einstein relation (2) the current components are obtained from the inverse Laplace-transformed diffusion coefficient

$$j_x(t) = \frac{e^2 E_x}{4\pi^2\hbar}\,\frac{1}{\alpha}\,f(t) \qquad j_y(t) = \frac{e^2 E_y}{4\pi^2\hbar}\,\alpha\, f(t) \tag{12}$$

where the dimensionless current density $f(t)$ is given by

$$f(t) = \frac{1}{2\pi \mathrm{i}}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}s}{s}\,\mathrm{e}^{st}\,D(s) \qquad f(s) = 4\pi^2\hbar N_F D(s). \tag{13}$$

It is expedient to introduce a new function $\varphi(t) = \exp(t/\tau_\varepsilon)\,\mathrm{d}f/\mathrm{d}t$, which is used to derive an explicit integral representation for the dimensionless current $f(t)$ via the following expression:

$$f(t) = f_\infty - \int_{t/t_0}^{\infty} \mathrm{d}x\;\exp(-xt_0/\tau_\varepsilon)\,\varphi(x) \tag{14}$$

where a characteristic time $t_0 = 4\pi^2\hbar N_F/\kappa^2\varepsilon_0^2$ has been introduced. In (14) $f_\infty = f(s = 0)$ is the dimensionless stationary current, which is obtained from the following transcendental equation:

$$f_\infty = \ln\left[A\,\frac{\sqrt{1+\varepsilon^2+\varepsilon_0^2\,t_0/(f_\infty\tau_\varepsilon)}-1}{\sqrt{1+\varepsilon^2+\varepsilon_0^2\,t_0/(f_\infty\tau_\varepsilon)}+1}\right] \tag{15}$$

with $A = \exp(2\Theta)$. The desired explicit solution of the self-consistent equation for the dimensionless current $f(t)$ is obtained by inverting the function $\varphi(s)$ and introducing the new integration variable $z = \exp(\varphi)$:

$$\varphi(x) = \frac{\mathrm{i}}{2\pi x}\int \frac{\mathrm{d}z}{z}\exp\left(x\ln z\left[\frac{4Az}{\varepsilon^2(A-z)^2} - (\varepsilon/\varepsilon_0)^2\right]\right). \tag{16}$$

The long-time behaviour of the current relaxation is obtained from this representation by applying the method of steepest descent:

$$f(t) = f_\infty + \frac{2}{B_1}\left\{\frac{1}{\sqrt{t/T}}\exp\left[-\frac{t}{T}\left(B_2+\frac{T}{\tau_\varepsilon}\right)\right] - \sqrt{\pi\left(B_2+\frac{T}{\tau_\varepsilon}\right)}\,\mathrm{erfc}\left[\sqrt{\frac{t}{T}\left(B_2+\frac{T}{\tau_\varepsilon}\right)}\right]\right\} \tag{17}$$

where we have introduced the following characteristic decay time:

$$T = t_0\coth\Theta = \frac{2\pi^2\hbar N_F}{\kappa^2}\sinh 2\Theta\,. \tag{18}$$

The coefficients B_1 and B_2 are calculated from

$$B_1 = \left\{4\pi\sinh 2\Theta\,\frac{Az_0}{(A-z_0)^4}[2(A^2-z_0^2)+(z_0^2+4Az_0+A^2)\ln z_0]\right\}^{1/2} \tag{19}$$

and

$$B_2 = 2\sinh 2\Theta \, \frac{A z_0 (A + z_0)}{(A - z_0)^3} \ln^2 z_0 \tag{20}$$

where the saddle point position z_0 is the solution of the following transcendental equation:

$$(A - z_0)^3 (\varepsilon/\varepsilon_0)^2 = 4 A z_0 \{A - z_0 + (A + z_0) \ln z_0\} . \tag{21}$$

At the delocalization field strength ($\varepsilon/\varepsilon_0 = 1$), equation (21) has the solution $z_0 = 1$, which implies that $B_2 = 0$. Therefore, if inelastic scattering is neglected ($\tau_\varepsilon \to \infty$) one obtains from (17) a long-time power-law relaxation of the electric current according to $j(t) \sim 1/\sqrt{t}$. This result is in accordance with the fact that one-parameter scaling still persists [14, 15] and the critical exponents are independent of the anisotropy. The appearance of this long-time tail in the current relaxation is a characteristic feature of the metal–insulator phase transition, where near the critical point the relaxation processes rapidly slow down. If the electric field deviates only slightly from the edge field strength ($|E - E_0| \ll E_0$) the asymptotic representation (17) can be simplified further and one obtains

$$f(t) = f_\infty + \sqrt{\frac{T}{\pi t}} \exp\left(-\frac{t}{\tau_E}\right) - \sqrt{\frac{T}{\tau_E}} \operatorname{erfc}\left(\sqrt{\frac{t}{\tau_E}}\right) \tag{22}$$

where a second characteristic relaxation time

$$\frac{1}{\tau_E} = \frac{1}{T} \frac{(E - E_0)^2}{E_0^2} + \frac{1}{\tau_\varepsilon} \tag{23}$$

has been introduced. This result clearly demonstrates that the relaxation dynamics accelerates if the electric field deviates from the edge field strength or inelastic scattering processes are relevant.

4. Discussion

The anisotropy of the two-dimensional electron gas may be due to either an anisotropic effective mass or a tight-binding dispersion law along the y axis, which describes a system of parallel chains. In the former case, which was also treated by Kawabata [17], the energy spectrum has the form

$$\varepsilon(\boldsymbol{k}) = \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} . \tag{24}$$

Adopting this model of anisotropy the parameter α is given by $\alpha^2 = m_x/m_y = 1/\beta$ so the effective cut-off momentum takes the form $\kappa^2 = \sqrt{m_x m_y}/(4\varepsilon_F \tau_0^2)$, which implies according to (8)

$$\varepsilon^2 = \frac{(eE\tau_0)^2}{\varepsilon_F} \left(\frac{\cos^2\phi}{m_x} + \frac{\sin^2\phi}{m_y} \right) . \tag{25}$$

Under the condition of weak localization ($\varepsilon_F \tau_0/\hbar \gg 1$) the delocalization field strength E_0 given by

$$eE_0\tau_0 = \left[\varepsilon_F \left(\frac{\cos^2\phi}{m_x} + \frac{\sin^2\phi}{m_y} \right)^{-1} \right]^{1/2} \frac{1}{\sinh(\pi \varepsilon_F \tau_0/\hbar)} \tag{26}$$

depends exponentially on the disorder parameter $\varepsilon_F\tau_0/\hbar$. The same exponential dependence results also for an isotropic two-dimensional electron gas [13] (cf. also [18]). The only difference is that now the prefactor depends on the alignment of the electric field.

The time dependence of the current is determined by (17), where for weakly localized electrons ($\Theta \gg 1$) the parameters B_1 and B_2 are given by

$$B_1 = (2\pi z_0(2+\ln z_0))^{1/2} \qquad B_2 = z_0 \ln^2 z_0 \tag{27}$$

and the saddle point position is the solution of the equation $(\varepsilon/\varepsilon_0)^2 = z_0(1+\ln z_0)$. The time dependence of the current decay in such anisotropic systems agrees completely with our former result on an isotropic 2D electron gas [13].

More interesting are two-dimensional systems, where the anisotropy is due to a grid of parallel quantum wires. In this case the motion of electrons from chain to chain in the perpendicular direction can be described by a tight-binding dispersion law with the mini-band width w

$$\varepsilon(k) = \frac{\hbar^2 k_x^2}{2m_x} + w(1-\cos k_y a) \tag{28}$$

where we assume that only one mini-band is occupied. The wires are separated by the distance a. If the mini-band width is much larger than the Fermi energy ($w \gg \varepsilon_F$) we recover the model discussed above with $m_y = \hbar^2/wa^2$. More interesting is the case of narrow mini-bands ($w \ll \varepsilon_F$), where the density of states at the Fermi level $N_F = N_F^{(1)}/a$ is expressed by its 1D value $N_F^{(1)} = 1/\pi\hbar v_F$ and where the bare diffusion coefficients are given by $D_{0x} = \tau_0 v_F^2$, $D_{0y} = \tau_0 w^2 a^2/2\hbar^2$ and consequently $\Theta = \sqrt{2}\pi w\tau_0/\hbar$. Accordingly, we obtain for the anisotropy parameter $\alpha = wa/(\sqrt{2}\hbar v_F)$. The momentum cut-offs along the x and y directions are now quite different. Whereas the coherent transport along the x direction requires the cut-off $1/\kappa_x = \tau_0\sqrt{\langle v_x^2\rangle} = \tau_0 v_F$, there are two values for the perpendicular motion, namely $1/\kappa_y = \tau_0\sqrt{\langle v_y^2\rangle} = \tau_0 wa/\sqrt{2}\hbar$ and $1/\kappa_y = a/\pi$, the minimum of which has to be chosen. The crossover between a steep exponential and a power-law dependence of the delocalization edge is governed by the parameter Θ, which includes the mini-band width w and the elastic scattering time τ_0. For $\Theta < 1$ we obtain from (8) and (10) the following position of the edge field strength

$$\frac{eE_0\lambda_0}{\varepsilon_F} = 4\Bigg/\left[\left(1+\frac{2}{\Theta}\right)\left(\cos^2\phi + \left(\frac{a}{2\pi\lambda_0}\Theta\right)^2 \sin^2\phi\right)^{1/2} \sinh\Theta\right] \tag{29}$$

which is also a good approximation for $\Theta > 1$. Numerical results, illustrating this crossover for an electric field which is parallel or perpendicular to the chains, are shown in figure 1 for $a/(2\pi\lambda_0) = 0.1$. On the one hand, if $\Theta > 1$ the exponential dependence according to (26) is reproduced, where ε_F has to be replaced by $\sqrt{2}w$. On the other hand, if the mini-band width is smaller than the energy uncertainty due to scattering processes ($\Theta \ll 1$) then the motion of electrons from chain to chain along the y direction is in the regime of narrow-band transport. In this case the delocalization edge approaches the constant value $eE_{x0} = 4\varepsilon_F/(v_F\tau_0)$, which is the exact result of a one-dimensional chain [11] (cf also [12]) in a parallel electric field. In anisotropic samples with $\Theta < 1$ the edge field strength strongly depends on the alignment of the electric field, as can be seen from figure 1.

The time dependence of the current is obtained from (17), (19) and (20), where in the limit $\Theta \to 0$ the position of the saddle point $z_0 = 1 - 2\Theta(\rho_0 - 1)$ is calculated from $(\varepsilon/\varepsilon_0)^2\rho_0^3 + \rho_0 - 2 = 0$, implying $B_1 = \left[2\pi(3-\rho_0)/(\Theta\rho_0^4)\right]^{1/2}$ and $B_2 = 4\Theta(\rho_0-1)^2\rho_0^{-3}$, which completely agrees with the result of the one-dimensional case [13]. In the limit

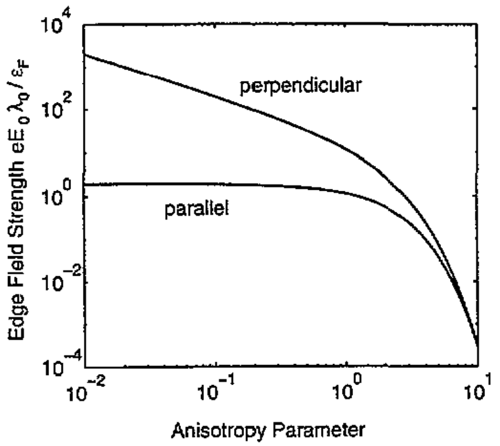

Figure 1. Dimensionless edge field strength $eE_0\lambda_0/\varepsilon_F$ as a function of the anisotropy parameter Θ for $a/(2\pi\lambda_0) = 0.1$. The electric field is aligned parallel and perpendicular to the chains.

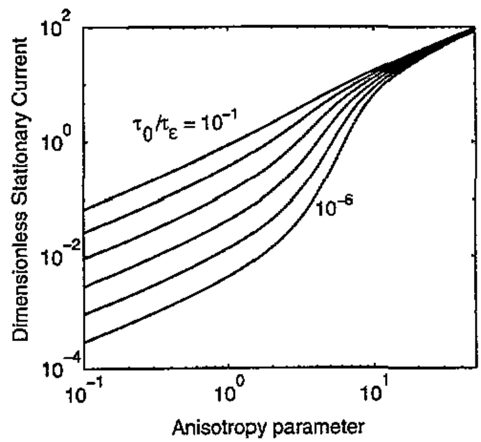

Figure 2. Dimensionless stationary current f_∞ as a function of the anisotropy parameter Θ calculated from (15). From bottom to top, the scattering time ratio is: $\tau_0/\tau_\varepsilon = 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$ and 10^{-1}.

$w \to 0$ the characteristic decay time T remains finite and approaches the value $T = 8\tau_0$. The dimensionless stationary current f_∞ exhibits a sharp increase in the transition region between the 1D and 2D limits. Numerical results obtained from (15) are shown in figure 2. Such an edgelike increase occurs only for sufficiently large inelastic scattering times $\tau_0/\tau_\varepsilon \ll 1$. In the crossover region $\Theta \sim 1$ the current–voltage characteristics become highly non-linear.

5. Conclusion

On the basis of a quantum diffusion model and using the Einstein relation for the mobility we calculated the direct influence of a static electric field on the quantum interference effects and the related current relaxation for an anisotropic two-dimensional electron gas. The anisotropic system is characterized by three characteristic energies—ε_F and $w_{x,y}$, where $w_{x,y}$ is the mini-band width along the x and y axis, respectively. The ratio of these energy parameters determines the behaviour of the system. Our approach predicts the existence of a delocalization edge, the field strength of which is proportional to $1/\sinh(bW\tau_0/\hbar)$, where b is some numerical constant and $W = \min(\varepsilon_F, w_x, w_y)$. If the band widths are larger than the Fermi energy ($w_x, w_y > \varepsilon_F$) the anisotropy can be described using the model of anisotropic effective masses, in which the anisotropy is weak and the delocalization edge is exponentially small $E_0 \sim \exp(-\pi\varepsilon_F\tau_0/\hbar)$, similarly to the case of weakly localized electrons in isotropic two-dimensional systems [12]. On the other hand, if one of the mini-bands is narrow, say $w_x > \varepsilon_F > w_y$, then the delocalization edge E_0 is proportional to $1/\sinh(\sqrt{2}\pi w_y\tau_0/\hbar)$. If the mini-band width w_y is smaller than $\hbar/\tau_0$, one observes a crossover from exponentially small edge field values, characteristic for 2D systems, to a much higher value, which approaches the constant 1D edge field strength. This crossover gives rise to highly non-linear current–voltage characteristics and has implications on the acceleration of current relaxation processes. Since the localization range is usually much larger in two dimensions than in one dimension, the effect of the delocalizing mechanisms is correspondingly greater. It is in line with the one-parameter scaling assumption that asymptotically and at $\tau_\varepsilon \to \infty$ the current relaxation has the power-law dependence $f(t) \sim 1/\sqrt{t}$ independently of the anisotropy.

In contrast to those in an isotropic two-dimensional system, the electric field effects should be more easily measurable in structured strongly anisotropic electron gases, because the edge field strength is much higher and the characteristic decay time T does not grow exponentially. Measurements which verify our predictions concerning the direct influence of a constant electric field on weak localization require low carrier densities and extremely low lattice temperatures.

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